Process-specific constraint effects in BMRS

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1 Introduction

This squib demonstrates how process-specific constraint (henceforth PSC) phenomena can be captured with *boolean monadic recursive schemes* (BMRS; Bhaskar et al., 2020), a logical formalism for analyzing phonological maps that is grounded in the computational nature of phonological generalizations. It presents a case study of one such effect in the RTR harmony system of Palestinian Arabic (Davis, 1995; McCarthy, 1997); rightward RTR harmony can be blocked by high, front segments, but leftward RTR harmony proceeds unimpeded within a word. This is shown below, the extent of the RTR feature is underlined.

(1) a. *Leftward harmony*: / ballas / → [ballas] 'thief'

b. *Rightward harmony*: / tuubak / → [tuubak] 'your blocks'

c. *Blocking of rightward harmony*: / sayyad / → [sayyad] 'hunter'

Davis (1995)'s rule-based analysis achieves the PSC effect by tagging the rightward spread rule with the target condition "RTR/Hi and RTR/Fr" such that segments with these features block rightward spread but not leftward spread. The additional claim is that OT does not predict these effects. McCarthy (1997) responds by showing that OT not only captures PSCs—a direct result of constraint ranking—but it also presents a more restrictive theory of PSCs. The transitive nature of ranking predicts that if some crucial

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ranking between markedness constraints produces a blocking effect for one process, the same effect will be observed for any other process compelled by a markedness constraint lower in the hierarchy. This principle is termed the *Subset Criterion*, and no such prediction is made by PSCs tagged on individual rules.

As shown below, analyses in BMRS capture phonological generalizations through ordered hierarchies of *licensing* and *blocking structures* (Chandlee and Jardine, 2020). Following McCarthy (1997)'s generalization about rankings for PSC interactions in OT grammars, the analysis presented here shows that BMRS analyses produce the same effects. These obtain both when the interaction is defined as a single 'combined map' analysis capturing the full grammar and for the composition of individual processes. Crucially, this is because hierarchical relations are preserved under composition. It also means that the Subset Criterion is predicted by this basic mechanism in BMRS in much the same way as it is predicted in OT as a result of ranking transitivity. This thus is an argument in favor of BMRS, whose restrictive computational properties avoid the typological overgeneration of OT analyses of spreading (see Chandlee and Jardine, 2020).

2 Spreading in BMRS

The BMRS formalism describes underlying and surface structures in terms of monadic (=unary), boolean functions that take segments as their domain. For example, RTR(x) is a function that returns $\top$ (true) when x is an RTR segment in the input, $\bot$ (false) otherwise. We can then describe a phonological process affecting the output value of the RTR feature by creating a definition for an *output* function RTR$'(x)$ that describes the conditions under which x is RTR in the output (i.e., when RTR$'(x)$ evaluates to $\top$) and when it is not (RTR$'(x)$ evaluates to $\bot$). For example, the BMRS definition of RTR$'(x)$ below describes leftward spread of an RTR feature.

(2) $$\text{RTR}'(x) \quad = \quad \texttt{if}\ \text{RTR}'(s(x))\ \texttt{then}\ \top\ \texttt{else}\ \text{RTR}(x)$$

The right-hand side of (2) reads, "if RTR$'(s(x))$ is true then return $\top$, otherwise return the value of RTR(x)," where $s(x)$ refers to the *s*uccessor of x (i.e., the immediately following segment). An example evaluation of (2) is given below for (1a) /balla$\underline{\text{s}}$/ $\rightarrow$ [$\underline{\text{ballas}}$].

(3)

	b	a	l	l	a	$\underline{\text{s}}$
RTR$'(s(x))$	$\top$	$\top$	$\top$	$\top$	$\top$	$\bot$
RTR(x)	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\top$
RTR$'(x)$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$
	$\underline{\text{b}}$	$\underline{\text{a}}$	$\underline{\text{l}}$	$\underline{\text{l}}$	$\underline{\text{a}}$	$\underline{\text{s}}$

The input values for RTR(x) are given in the first row; this is true only for segment 6 (/$\underline{\text{s}}$/), as it is the only underlyingly RTR segment. Thus, while RTR$'(s(x))$ evaluates to $\bot$ for 6 (as it has no successor), it evaluates to $\top$ for the 'default' RTR(x) in the `else` statement. Thus, RTR$'(x)$ evaluates to evaluates to $\top$ for 6. Thus when segment 5 is evaluated against the definition, it satisfies the recursively-defined RTR$'(s(x))$ and is output with the RTR feature. Further evaluation proceeds in an identical manner; thus, the output RTR span extends leftward to the edge of the word.

In this way, the BMRS formalism captures changes to the input by evaluation of boolean properties of segments, which potentially can be recursively evaluated. We call a property $\mathbb{P}(x)$ a *licensing structure* if, as with RTR$'(s(x))$, it is in the configuration `if` $\mathbb{P}(x)$ `then` $\top$, as it causes the statement to be returned true. In contrast, we call a property $\mathbb{P}(x)$ a *blocking structure* if it is in the configuration `if` $\mathbb{P}(x)$ `then` $\bot$, as it causes the statement to be returned false. An example will be seen below, in the blocking structure used to stop rightward spread. The power of the full BMRS formalism, then, lies in the interaction of these licensing and blocking structures. How these licensing and blocking structures are supplied is not the concern of this paper; like constraints in OT, for our purposes we can assume they are supplied by the universal grammar. However, do note that these structures can refer to segments local to x using $s(x)$ or $p(x)$, referring to the *p*redecessor of x.

3 PSC effects in a full BMRS grammar

In OT, the Arabic data motivate the following ranking of constraints, where RTR-LEFT/RIGHT triggers leftward/rightward spread and RTR/HI&FR captures the blocking condition.

(4) RTR-LEFT $\gg$ RTR/HI&FR $\gg$ RTR-RIGHT $\gg$ IDENT-RTR

In general, a PSC interaction obtains when some constraint $\mathbb{C}$ ranks between two markedness constraints $\mathbb{M}_i$ and $\mathbb{M}_j$, each of which outrank some faithfulness constraint $\mathbb{F}$.

Given $\mathbb{M}_i \gg \mathbb{C} \gg \mathbb{M}_j \gg \mathbb{F}$, the effect of $\mathbb{C}$ is 'specific' to the process triggered by $\mathbb{M}_j \gg \mathbb{F}$ and not to the one triggered by $\mathbb{M}_i$. This is apparent in the ranking in (4), where RTR/HI&FR is specific to the spreading motivated by RTR-RIGHT, but not that motivated by RTR-LEFT.

The PSC effects observed for Palestinian Arabic can be captured as a single BMRS definition.[2] Here, we add HI&FR as a blocking condition on RTR in the output, and situate it within the hierarchy in (5).

(5)
$$\begin{aligned}\text{RTR}'(x) \;=\; & \texttt{if } \text{RTR}'(s(x)) \quad \texttt{then } \top \texttt{ else} \\ & \texttt{if } \text{HI\&FR}(x) \quad \texttt{then } \bot \texttt{ else} \\ & \texttt{if } \text{RTR}'(p(x)) \quad \texttt{then } \top \texttt{ else } \text{RTR}(x)\end{aligned}$$

Rightward spread obtains via initial satisfaction of RTR(x) followed by iterative evaluation of RTR$'(p(x))$. However, since the blocking structure HI&FR(x) comes before RTR$'(p(x))$ in the hierarchy, spreading can only proceed provided the current input symbol does not return a 'true' value for HI&FR(x). If it does, then spreading is blocked as in (1c), whose output is computed against the system below.

[2]See (Oakden et al., 2020) for proof that BMRS systems which call both predecessor and successor functions are equivalent to the regular class of functions.

(6)

	s̲	a	y	y	a	d
$\textsc{Hi\&Fr}(x)$	⊥	⊥	⊤	⊤	⊥	⊥
$\text{RTR}'(p(x))$	⊥	⊤	⊤	⊥	⊥	⊥
$\text{RTR}(x)$	⊤	⊥	⊥	⊥	⊥	⊥
	s̲	a̲	y	y	a	d

RTR ‘spreads’ to segment 2 from the trigger /s̲/, but since segment 3 satisfies the higher-ranked blocking structure (in spite of also evaluating to true for $\text{RTR}'(p(x))$), it returns ‘false’ for $\text{RTR}'(x)$ and blocks further rightward spread. Importantly, the same hierarchical stratification that blocks rightward spread also permits leftward spread over high, front segments, as shown below with /xayyat̲/→[x̲a̲y̲y̲a̲t̲].

(7)

	x	a	y	y	a	t̲
$\text{RTR}'(s(x))$	⊤	⊤	⊤	⊤	⊤	⊥
$\textsc{Hi\&Fr}(x)$	⊥	⊥	⊤	⊤	⊥	⊥
$\text{RTR}(x)$	⊥	⊥	⊥	⊥	⊥	⊤
	x̲	a̲	y̲	y̲	a̲	t̲

In spite of returning a ‘true’ value for $\textsc{Hi\&Fr}(x)$, segments 3 and 4 surface with an RTR feature by virtue of satisfying the licensing structure $\text{RTR}'(s(x))$ higher in the hierarchy, allowing the span to spread to the beginning of the word. This hierarchy and the observed PSC blocking mirror the constraint ranking posited by McCarthy.

4 PSC effects in a composite system

PSC effects are captured in a combined map BMRS system of equations mirroring McCarthy's constraint hierarchy; they also persist when each spreading process is defined as a separate system and the two combine through composition, more akin to Davis' rule-based account. Importantly, this squib does not decide between the two, but instead demonstrates that both preserve the Subset Criterion when formalized using BMRS. Let some system of equations L model unhindered leftward RTR spreading. Its output boolean function $\text{RTR}_L(x)$ is defined below.

(8) $\text{RTR}_L(x) \;=\; \texttt{if } \text{RTR}_L(s(x)) \texttt{ then } \top \texttt{ else } \text{RTR}(x)$

Similarly, let a system R denote a BMRS system of equations modeling rightward spreading with the blocking condition. Note that the blocking condition supersedes the licensing condition in the hierarchy; this produces the effect of rightward spreading which is blocked by any high, front segment.

(9) $\text{RTR}_R(x) \;=\; \texttt{if } \text{H}\textsc{i}\&\text{F}\textsc{r}(x) \texttt{ then } \bot \texttt{ else}$

$\texttt{if } \text{RTR}_R(p(x)) \texttt{ then } \top \texttt{ else } \text{RTR}(x)$

Now let $L \circ R$ be the *composition*[3] of these systems such the structure of L is preserved, and where each non-recursively defined boolean function in that system refers to the corresponding function name definition in system R.

(10) $\text{RTR}_L(x) \;=\; \texttt{if } \text{RTR}_L(s(x)) \texttt{ then } \top \texttt{ else } \text{RTR}_R(x)$

[3]For a proof that this operation is equivalent to function composition, see (Oakden et al., 2020)

The reader can confirm that the composite system is extensionally equivalent to the combined map system in (5), and computes the same set of mappings, e.g. (3), (6), and (7). Computation of an output string proceeds first through the portion of the hierarchy responsible for leftward spread (RTR$_L(x)$), then through the established hierarchy for rightward spread (RTR$_R(x)$). Since composition does not alter existing hierarchical relations within individual systems, we may say that hierarchies are preserved under composition.

5 BMRS preserves the Subset Criterion

One direct consequence of hierarchical relations in BMRS systems of equations and their preservation under composition is that McCarthy/Prince's Subset Criterion falls out automatically. McCarthy presents a general schema for this criterion, where $\mathbb{L}$ represents a constraint imposing a specific limitation on $\mathbb{M}_i$ (which, recall, is the markedness constraint compelling some process not influenced by blocker $\mathbb{C}$):

(11) $\mathbb{L} \gg \mathbb{M}_i \gg \mathbb{C} \gg \mathbb{M}_j \gg \mathbb{F}$

The Subset Criterion is thus derived from the transitive ranking of constraints: "if $\mathbb{M}_i \gg \mathbb{M}_j \gg \mathbb{F}$, then the set of constraints that can, in principle, impinge on $\mathbb{M}_i$ is a subset of the set of constraints that can, in principle, impinge on $\mathbb{M}_j$" (239). In other words, when higher-ranked $\mathbb{M}_i$ is subject to a PSC, lower-ranked $\mathbb{M}_j$ may also be subject to that PSC.

Hierarchies of licensing and blocking structures and their preservation under composition yields the same effect in the BMRS formalism. A simplified example using

McCarthy's notation demonstrates this fact. Let some system F model a process licensed by a structure $\mathbb{M}_i$ but which is subject to some (output-oriented) PSC limitation $\mathbb{L}$. A relevant output boolean function $\mathrm{A}_F(x)$ is defined where the PSC is ordered before the licenser.

(12)
$$\begin{aligned} \mathrm{A}_F(x) \;=\; & \texttt{if } \mathbb{L}(x) \;\; \texttt{then } \bot \texttt{ else} \\ & \texttt{if } \mathbb{M}_i(x) \texttt{ then } \top \texttt{ else } \mathrm{A}(x) \end{aligned}$$

Similarly, let system G model a process licensed by a structure $\mathbb{M}_j$ but subject to some (also output-oriented) PSC limitation $\mathbb{C}$. An equivalent definition for $\mathrm{A}_G(x)$ is as follows.

(13)
$$\begin{aligned} \mathrm{A}_G(x) \;=\; & \texttt{if } \mathbb{C}(x) \;\; \texttt{then } \bot \texttt{ else} \\ & \texttt{if } \mathbb{M}_j(x) \texttt{ then } \top \texttt{ else } \mathrm{A}(x) \end{aligned}$$

Composition $F \circ G$ follows by the normal mechanism; in this case the default condition $\mathrm{A}(x)$ in system F—the only non-recursively defined boolean function—is indexed with the corresponding definition from G. Importantly, all hierarchical relations between licensing and blocking structures are preserved. The result is that F's PSC limitation $\mathbb{L}(x)$ is calculated before $\mathbb{M}_i(x)$ given the hierarchy, and necessarily before $\mathbb{M}_j$ (in system G) given the hierarchical relation between $\mathbb{L}(x)$ and $\mathrm{A}(x)$ in system F. In other words, when $\mathbb{M}_i$ is subject to $\mathbb{L}$, so is $\mathbb{M}_j$. An equivalent system illustrates the full hierarchy.

(14)
$$\begin{aligned} \mathrm{A}_F(x) \;=\; & \texttt{if } \mathbb{L}(x) \;\; \texttt{then } \bot \texttt{ else} \\ & \texttt{if } \mathbb{M}_i(x) \texttt{ then } \top \texttt{ else} \\ & \texttt{if } \mathbb{C}(x) \;\; \texttt{then } \bot \texttt{ else} \\ & \texttt{if } \mathbb{M}_j(x) \texttt{ then } \top \texttt{ else } \mathrm{A}(x) \end{aligned}$$

This reflects precisely the total order in (11), and produces the same effects. Thus, the hierarchical relations and their preservation under composition in BMRS follows the same irreflexive, asymmetric, and transitive nature of the strict ordering relation over OT constraints, a property not derived by rule-based accounts with PSC tags on individual rules.

6 BMRS avoids pathological PSC effects

By McCarthy's account, the Subset Criterion—driven by the basic mechanism of constraint interaction—results in a more restrictive theory of PSC than is available to the rule-based formalism. Davis (1995) posits a hypothetical harmony system where rightward spread is subject to one condition and leftward spread is subject to a different condition. Such a case is predicted to be impossible in OT because it would require a circular ranking. McCarthy illustrates with a toy example using the de-conjoined RTR/HI and RTR/FR as separate conditions on rightward and leftward spread. The required rankings are thus (240):

(15)		Ranking	Interpretation
	a.	RTR/HI $\gg$ RTR-RIGHT	High segments block rightward harmony.
	b.	RTR-RIGHT $\gg$ RTR/FR	Front segments don't block rightward harmony.
	c.	RTR/FR $\gg$ RTR-LEFT	Front segments block leftward harmony.
	d.	RTR-LEFT $\gg$ RTR/HI	High segments don't block leftward harmony.

A total order maintaining these sub-rankings is impossible; RTR/HI cannot rank above

RTR-RIGHT *and* below RTR-LEFT when RTR-RIGHT $\gg$ RTR-LEFT via transitivity.

BMRS systems of equations make the same predictions about the hypothetical case above and thus align with the restrictions on PSC imposed by the Subset Criterion. To see how, consider two systems of equations R and L modeling rightward and leftward spreading with separate PSC conditions:

(16) a. $\text{RTR}_R(x) = \texttt{if } \text{RTR/HI}(x) \texttt{ then } \bot \texttt{ else}$
$\texttt{if } \text{RTR}_R(p(x)) \texttt{ then } \top \texttt{ else } \text{RTR}(x)$

b. $\text{RTR}_L(x) = \texttt{if } \text{RTR/FR}(x) \texttt{ then } \bot \texttt{ else}$
$\texttt{if } \text{RTR}_L(s(x)) \texttt{ then } \top \texttt{ else } \text{RTR}(x)$

Preservation of hierarchical relations under composition guarantees that every composite system definable from single systems will meet the Subset Criterion, and so also predicts that no BMRS system of equations can describe the hypothetical grammar in (15). Instead, the possible compositions $R \circ L$ and $L \circ R$ for the example above maintain the subset/superset relationship between conditions on two spreading patterns.

However, BMRS preserves the Subset Criterion for PSC effects differently from OT. For example, while in OT it is impossible to produce a hierarchy with the subrankings in (15), it is possible to define separate BMRS systems of equations with these exact relations intact, and then compose them to form a full grammar. The systems in (17) append the definitions in (16) to include *all* subrankings from McCarthy's pathological hierarchy:

(17) a. $\text{RTR}_R(x) \;=\; \texttt{if } \text{RTR/H}\textsc{i}\,(x) \texttt{ then } \bot \texttt{ else}$

$\texttt{if } \text{RTR}_R(p(x)) \texttt{ then } \top \texttt{ else}$

$\texttt{if } \text{RTR/F}\textsc{r}(x) \texttt{ then } \bot \texttt{ else } \text{RTR}(x)$

b. $\text{RTR}_L(x) \;=\; \texttt{if } \text{RTR/F}\textsc{r}(x) \texttt{ then } \bot \texttt{ else}$

$\texttt{if } \text{RTR}_L(s(x)) \texttt{ then } \top \texttt{ else}$

$\texttt{if } \text{RTR/H}\textsc{i}(x) \texttt{ then } \bot \texttt{ else } \text{RTR}(x)$

Composing these systems in either direction yields a (somewhat redundant) system that *still observes the Subset Criterion*, because of preservation of hierarchical relations under composition. A definition equivalent to $R \circ L$ is given below; high segments block rightward *and* leftward spreading while front segments *only* block leftward spreading.

(18) $\text{RTR}'(x) \;=\; \texttt{if } \text{RTR/H}\textsc{i}(x) \texttt{ then } \bot \texttt{ else}$

$\texttt{if } \text{RTR}'(p(x)) \texttt{ then } \top \texttt{ else}$

$\texttt{if } \text{RTR/F}\textsc{r}(x) \texttt{ then } \bot \texttt{ else}$

$\texttt{if } \text{RTR/F}\textsc{r}(x) \texttt{ then } \bot \texttt{ else}$

$\texttt{if } \text{RTR}'(s(x)) \texttt{ then } \top \texttt{ else}$

$\texttt{if } \text{RTR/H}\textsc{i}(x) \texttt{ then } \bot \texttt{ else } \text{RTR}(x)$

Thus unlike OT, the BMRS formalism *can* capture Davis' hypothetical systems where individual 'rules' (in this case separate systems of equations) are subject to distinct conditions. Unlike Davis' rule-based conception, though, the composition of those systems necessarily obeys the Subset Condition. This is a direct consequence of two basic components of BMRS systems of equations: hierarchies of licensing and blocking structures, and their preservation under composition.

References

Bhaskar, S., Chandlee, J., Jardine, A., and Oakden, C. (2020). Boolean monadic recursive schemes as a logical characterization of the subsequential functions. In Leporati, A., Martín-Vide, C., Shapira, D., and Zandron, C., editors, *Language and Automata Theory and Applications - LATA 2020*, Lecture Notes in Computer Science, pages 157–169. Springer.

Chandlee, J. and Jardine, A. (2020). Recursive schemes for phonological analysis. Unpublished ms., Haverford College and Rutgers University. Available at `http://adamjardine.net/files/chandleejardineBMRSms.pdf`.

Davis, S. (1995). Emphasis spread in Arabic and grounded phonology. *Linguistic Inquiry*, 26:465–498.

McCarthy, J. J. (1997). Process-specific constraints in Optimality Theory. *Linguistic Inquiry*, 28:231–251.

Oakden, C., Bhaskar, S., and Jardine, A. (2020). Some further properties of BMRS transductions. Unpublished ms., Rutgers University and Københavns Universitet. Available at `https://chrisoakden.com/files/Further_properties_BMRS.pdf`.